



NATIONAL OPEN UNIVERSITY OF NIGERIA
PLOT 91, CADASTRAL ZONE, NNAMDI AZIKIWE EXPRESSWAY, JABI - ABUJA
FACULTY OF SCIENCES

DEPARTMENT OF PURE AND APPLIED SCIENCE

2023_2 EXAMINATIONS

COURSE CODE: PHY312

COURSE TITLE: MATHEMATICAL METHODS FOR PHYSICS II

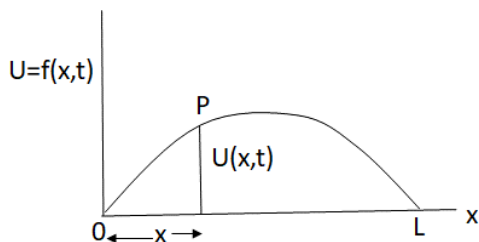
CREDIT UNIT: 3

TIME ALLOWED: (2½ HRS)

INSTRUCTION: Answer question 1 and any other four questions

QUESTION 1

A (i). Consider a perfectly flexible elastic string stretched between two points at $x=0$ and $x=l$ (see Figure below) with uniform tension T . If the string is displaced slightly from its initial position of rest and released, with the end point remaining fixed, then the string will vibrate. The position of any point P in the string will then depend on its distance from one end and on one instant in time. Given that its displacement $u=f(x,t)$; where x is the distance from the left hand, write the equations of motion in terms of X and T using the separation of variables approach. [6 marks]



(ii). If a wave equation is given as $\nabla^2 u = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$; what are the physical parameters represented by u and v ? [2marks]

(iii). Find the complete solution of $pq = xy$ [5marks]

B. Evaluate the integral $\int \frac{dx}{6x^2 - 5x + 1}$ [5marks]

C. Solve the equation $(X^2 - 4XX' + 3X'^2)b = 0$ [4marks]

QUESTION 2

A. Solve the equation $\frac{\partial^2 u}{\partial x \partial y} = \sin(x + y)$ given that at $y = 0$, $\frac{\partial u}{\partial x} = 1$ and at $x=0$ $u = (y - 1)^2$

[7marks]

B. Solve $\frac{\partial z}{\partial x} = \sin x$, for $z(x,y)$

[5marks]

QUESTION 3

A. Find the complete integral of the equation $P^2x + q^2y = Z$ using Jacobis method. [6marks]

B(i). Give one advantage of using Jacobis method for solving integration. [2marks]

(ii). Determine the inverse Laplace transform of $\frac{1}{(s-2)(s^2+1)}$. [4marks]

QUESTION 4

A. Find the tenth (a_{10}) Fourier coefficient of the Fourier series: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ [6marks]

B (i). State Bessel's differential equation. [2marks]

(ii). Given that $J_{-n}(x) = \sum_{r=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{-n+2r}}{r! \Gamma(-n+r+1)}$; What is the value for $J_{-\frac{1}{2}}(x)$? [4marks]

QUESTION 5

A. Find the Fourier Cosine series for $F(x)=e^x$ at $(0, \pi)$ [5marks]

B. If solution $y(x)$, of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{-x}$ is subjected to the boundary conditions $y(0) = 2$, $y'(0) = 1$ has the form $y(x) = Ae^{-x} + Be^x + Ce^{2x}$. Evaluate the value of $A + B - C$. [7marks]

QUESTION 6

A .Define a half-range Fourier series for a function $f(x)$; obtain a Fourier Cosine series which respects an even periodic Function $F_1(t)$ of period $T=2l$. [7marks]

B (i). Laplace and Fourier transforms are useful in solving a variety of partial differential equations; what would guide the choice of the appropriate transform? [2mark]

(ii). List at least three periodic phenomena in nature [3marks]